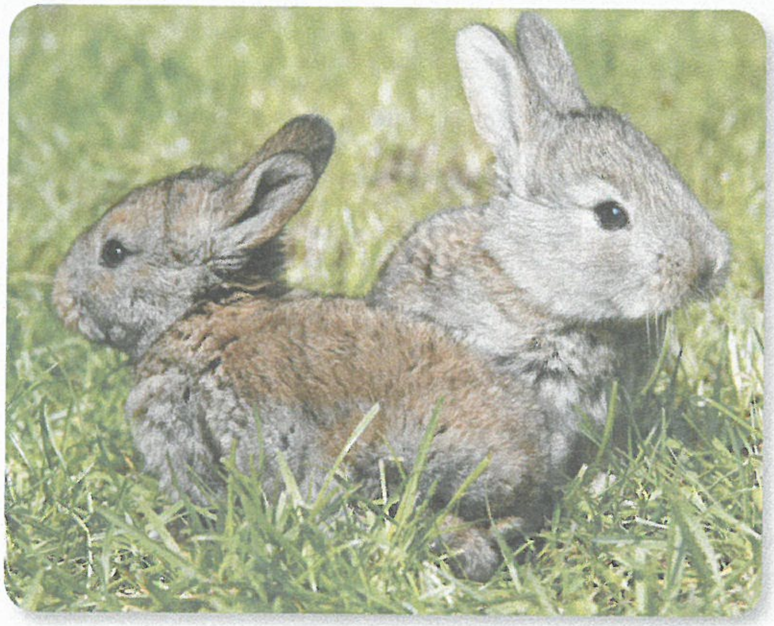
CHAPTER

1 Operations on Real Numbers and Algebraic Expressions



In the year 1202, the Italian mathematician Leonardo Fibonacci posed this problem: A certain man put a pair of rabbits in a place surrounded on all sides by a wall. How many pairs of rabbits can be produced from that pair in a year if every month each pair begets a new pair that is productive from the second month on?

The answer to Fibonacci's puzzle leads to a sequence of numbers called the *Fibonacci sequence*. See Problem 155 in Section 1.4.

The Big Picture: Putting It Together

Welcome to algebra! This course is taken by a diverse group of individuals. Some of you may never have taken an algebra course, while others may have taken algebra at some time in the past. In any case, we have written this text with both groups in mind.

The first chapter of the text reviews arithmetic. The material is presented with an eye on the future, which is algebra. This means that we will slowly build our discussion so that the shift from arithmetic to algebra is painless. Carefully study the methods used in this section, because these same methods will be used again in later chapters.

Outline

1.1 Success in Mathematics

Objectives

1. What to Do the First Week of the Semester
2. What to Do Before, During, and After Class
3. How to Use the Text Effectively
4. How to Prepare for an Exam

Let's start by having a discussion about the "big picture" goals of the course and how this text can help you to be successful at mathematics. Our first "big picture" goal is to develop algebraic skills and gain an appreciation for the power of algebra and mathematics. But there is also a second "big picture" goal. By studying mathematics, we develop a sense of logic and exercise the part of our brains that deals with logical thinking. The examples and problems in this text are like the crunches we do in a gym to exercise our bodies. The goal of running or walking is to get from point A to point B, so doing fifty crunches on a mat does not accomplish that goal, but crunches do make our upper bodies, backs, and hearts stronger when we need to run or walk.

Logical thinking can assist us in solving difficult everyday problems, and solving algebra problems "builds the muscles" in the part of our brain that performs logical thinking. So, when you are studying algebra and getting frustrated with the amount of work that needs to be done, and you say, "My brain hurts," remember the phrase we all use in the gym, "No pain, no gain."

Another phrase to keep in mind is "Success breeds success." Mathematics is everywhere. You already are successful at doing some everyday mathematics. With practice, you can take your initial successes and become even more successful. Have you ever done any of the following everyday activities?

- Compare the price per ounce of different sizes of jars of peanut butter or jam.
- Leave a tip at a restaurant.
- Figure out how many calories your bowl of breakfast cereal provides.
- Take an opinion survey along with many other people.
- Measure the distances between cities as you plan a vacation.
- Order the appropriate number of gallons of paint to cover the walls of a room.
- Buy a car and take out a car loan with interest.
- Double a cookie recipe.
- Exchange American dollars for Canadian dollars.
- Fill up a basketball or soccer ball with air (balls are spheres, after all).
- Coach a Little League team (scores, statistics, catching, and throwing all involve math).
- Check the percentages of saturated and unsaturated fats in a chocolate bar.

You may do five or ten mathematical activities in a single day! The everyday mathematics that you already know is the foundation for your success in this course.

1 What to Do the First Week of the Semester

The first week of the semester gives you the opportunity to prepare for a successful course. Here are the things you should do:

1. **Pick a good seat.** Choose a seat that gives you a good view of the room. Sit close enough to the front so you can easily see the board and hear the professor.
2. **Read the syllabus to learn about your instructor and the course.** Take note of your instructor's name, office location, e-mail address, telephone number, and office hours. Pay attention to any additional help available, such as tutoring centers, videos in the library, software, online tutorials, and so on. Be sure you fully understand all of the instructor's policies for the class, including the policy on absences, missed exams or quizzes, and homework. Ask questions.
3. **Learn the names of some of your classmates and exchange contact information.** One of the best ways to learn math is through group study sessions. Try to create time each week to study with your classmates. Knowing how to get in contact with classmates is also useful if you ever miss class, because you can obtain the assignment for the day.

Work Smart: Study Skills

Plan on studying at least two hours outside of class for each hour in class every week.

4. **Budget your time.** Most students have a tendency to "bite off more than they can chew." To help with time management, consider the following general rule: Plan on studying *at least* two hours outside of class for each hour in class. Thus, if you enrolled in a four-hour math class, you should set aside at least eight hours each week to study for the course. You will also need to set aside time for other courses. Consider your work schedule and personal life when creating your time budget.

2 What to Do Before, During, and After Class

Now that the semester is under way, we present the following ideas for what to do before, during, and after each class meeting. These suggestions may sound overwhelming, but we guarantee that by following them, you will be successful in mathematics (and other courses). Also, you will find that studying for exams becomes much easier by following this plan.

Before Class Begins

1. Read the section or sections that will be covered in the upcoming class meeting. Watch the video lectures that accompany the text.
2. Based on your reading, write down a list of questions. Your questions will probably be answered through the lecture. You can then ask any questions that are not answered completely.
3. Make sure you are mentally prepared for class. Your mind should be alert and ready to concentrate for the entire class. (Invest in a cup of coffee and eat lots of protein for breakfast!)

During Class

1. Arrive early enough to prepare your mind and material for the lecture.
2. Stay alert. Do not doze off or daydream during class. If you do so, understanding the lecture will be very difficult when you "return to class."
3. Take thorough notes. It is normal not to understand certain topics the first time you hear them in a lecture. However, this does not mean that you throw your hands up in despair. Rather, continue to take class notes.
4. Do not be afraid to ask questions. In fact, instructors love questions, for two reasons. First, if one student has a question, other students probably have the same question. Second, by asking questions, you teach the teacher what topics cause difficulty.

Work Smart: Study Skills

Be sure to ask questions during class.

After Class

1. Reread (and possibly rewrite) your class notes. In our experience as students, we were amazed how often our confusion during class disappeared after we studied our in-class notes after class.
2. Reread the section. This is an especially important step. Once you have heard the lecture, the section will make more sense and you will understand much more.
3. Do your homework. **Homework is not optional.** There is an old Chinese proverb that says,

   I hear … and I forget
   I see … and I remember
   I do … and I understand

   This proverb applies to any situation in life in which you want to succeed. Would a pianist expect to be the best if she didn't practice? The only way you are going to learn algebra is by doing algebra.
4. When you get a problem wrong, try to figure out why you got the problem wrong. If you can't discover your error, be sure to ask for help.

Work Smart: Study Skills

The reason for homework is to build your skill and confidence. Don't skip assignments.

5. If you have questions, visit your professor during office hours. You can also ask someone in your study group or go to the tutoring center on campus, if available.

Learning Is a Building Process

Learning is the art of making connections between thousands of neurons (specialized cells) in the brain. Memory is the ability to reactivate these neural networks—it is a conversation among neurons.

Math isn't a mystery. You already know some math. But you do have to practice what you know and expand your knowledge. Why? The brain contains thousands of neurons. Through repeated practice, signals in the brain travel faster. The cells "fire" more quickly, and connections are made faster and with less effort. Practice allows us to retrieve concepts and facts at test time. Remember those crunches, which are a way of making your body more robust and nimble—learning does the same to your brain.

Have We Mentioned Asking Questions?

To move information from short-term memory to long-term memory, we need to think about the information, comprehend its meaning, and ask questions about it.

3 How to Use the Text Effectively

When we sat down to write this text, we knew from our teaching experience that students typically do not read their math books. We decided to accept the steps students usually go through:

1. Attend the lecture and watch the instructor do some problems on the board. Perhaps work some problems in class.
2. Go home and work on the homework assignment.
3. After each problem, check the answer in the back of the text. If you were right, move on, but if wrong, go back and see where the solution went wrong.
4. If the mistake cannot be identified, go to your class notes or try to find a similar example in the text. With a little luck, the student can determine where the solution went wrong in the problem.
5. If Step 4 fails, mark the problem and ask about it in the next class meeting, which leads us back to Step 1.

With this model in mind, we started to develop this text so that there is more than one way to extract the information you need from it.

All of the features in the text are here to help you succeed. These features are based on techniques we use in class. The features that appear, an explanation of the purpose of each feature, and how each can be used to help you succeed in this course are outlined in the following paragraphs.

Are You Ready for This Section?: Warming Up

Each section (after Section 1.2) begins with a short "readiness quiz." This quiz asks questions about material that was presented earlier in the course and is needed for the upcoming section. Take the readiness quiz to be sure you understand the material that the new section is based on. Answers to the quiz appear in a footnote on the page of the quiz. Check your answers. If you get a problem wrong or don't know how to do a problem, go back to the section listed and review the material.

Objectives: A "Road Map" through the Course

To the left of the readiness quiz is a list of objectives to be covered in the section. If you follow the objectives, you will get a good idea of the section's "big picture"—the important concepts, techniques, and procedures.

The objectives are numbered. (See the numbered headline at the beginning of this section.) When we begin discussing a particular objective within the section, the objective number appears along with the stated objective.

Examples: Where to Look for Information

Examples are meant to provide you with guidance and instruction when you are away from the instructor and the classroom. With this in mind, we have developed two special example formats.

Step-by-Step Examples have a three-column format where the left column describes a step, the middle column briefly explains the step, and the right column presents the algebra. Thus the left and middle columns can be thought of as your instructor's voice during a lecture. *Step-by-Step Examples* introduce key topics or important problem-solving strategies. They provide easy-to-understand, practical instructions by including the words "how to" in the examples' title.

Annotated Examples have a two-column format with explanations to the left of the algebra. The explanation clearly describes what we are about to do in the order in which we will do it. Again, annotations are like your instructor's voice as he or she writes each step of the solution on the board.

Authors in Action: Lecture Videos to Help You Learn

Every objective has one or more classroom lecture videos of the authors teaching their students. These "live" classroom lectures can be used to supplement your instructor's presentations and your reading of the text. These videos can be found in the Multimedia Library of MyMathLab or on a DVD video series and are marked with an ▶ icon in the text.

In Words: Math in Everyday Language

Have you ever been given a math definition in class and said, "What in the world does that mean?" We have heard that from our students. So we added the "In Words" feature, which restates mathematical definitions in everyday language. This margin feature will help you understand the language of mathematics better. See page 11.

Work Smart

These "tricks of the trade" that appear in the margin can help you solve problems. They also show alternative problem-solving approaches. There is more than one way to solve a math problem! See page 9.

Work Smart: Study Skills

These margin notes highlight the study skills required for success in this and other mathematics courses. See page 7.

Exercises: A Unique Numbering Scheme

As teachers, we know that students typically jump right to the exercises after attending class. This means they tend to skip all of the examples and explanations of concepts in the section. To help you use the text most effectively to learn the math, we have structured the exercises differently from other texts you have used. Our structure is designed to encourage the reading of the text, while increasing your confidence and ability to work any mathematical problem. For this reason, the exercises in each section are broken into as many as seven parts. Each exercise set will have some, or all, of the following exercise types.

1. Quick Checks
2. Building Skills
3. Mixed Practice

4. Applying the Concepts
5. Extending the Concepts
6. Explaining the Concepts
7. The Graphing Calculator

1. **Quick Checks: Learning to Ride a Bicycle with Training Wheels** Do you remember when you were first learning to ride a bicycle? Training wheels were placed on the bicycle to assist you in learning balance. The Quick Checks are like exercises with training wheels. These exercises appear right after the example or examples that illustrate the concept being taught. So, if you get stuck on a Quick Check problem, you can simply consult the example immediately preceding it, rather than searching through the text. The Quick Check exercises also verify your understanding of new vocabulary. See Quick ✓on page 9 in Section 1.2.
2. **Building Skills: Learning to Ride a Bicycle with Assistance** Once you felt ready to ride without training wheels, you probably had an adult follow closely behind you, holding the bicycle for balance and building your confidence. The Building Skills problems serve a similar purpose. They are keyed to the objectives within the section, so the directions for the problem indicate which objective is being developed. As a result, you know exactly which objective (but not exactly which example) to consult if you get stuck. See page 16 in Section 1.2.
3. **Mixed Practice: Now You Are Ready to Ride!** After mastering training wheels and learning to balance with assistance, you are ready to ride alone. This stage corresponds to the Mixed Practice exercises. These exercises include problems that develop your ability to see the big picture of mathematics. They are not keyed to a particular objective and require you to determine the appropriate approach to solving a problem on your own. See page 17 in Section 1.2.
4. **Applying the Concepts: Where Will I Ever Use This Stuff?** The Applying the Concepts exercises not only illustrate the application of mathematics in your life but also provide problems that test your conceptual understanding of the mathematics. See pages 17–18 in Section 1.2.
5. **Extending the Concepts: Stretching Your Mind** Sometimes we need to be challenged. These exercises extend your skills to a new level and provide further insight into where mathematics can be used. See page 18 in Section 1.2.
6. **Explaining the Concepts: Verbalize Your Understanding** These problems require you to express the section's big-picture concepts in your own words. Students need to improve their ability to communicate complicated ideas (both oral and written). If you truly understand the material in the section, you should be able to articulate the concepts clearly. See page 27 in Section 1.3.
7. **The Graphing Calculator** The graphing calculator is a great tool for verifying answers and for helping to visualize results. These exercises illustrate how the graphing calculator can be incorporated into the material of the section. See page 259 in Section 4.1.

Chapter Review

The chapter review is arranged section by section. For each section, we state key concepts, key terms, and objectives. We also list the examples and page numbers from the text that illustrate each objective. Also, for each objective, we list the problems in the review exercises that test your understanding. If you get a problem wrong, use this feature to determine where to look in the book to help you to work the problem.

Chapter Test

We have included a chapter test. Once you think you are prepared for the exam, take the chapter test. If you do well on the chapter test, chances are you will do well on your in-class exam. Be sure to take the test under the conditions you will face in class. If you are unsure how to solve a problem in the chapter test, watch the Chapter Test Prep Videos, which shows an instructor solving each chapter test problem.

Cumulative Review: Reinforcing Your Knowledge

The building process of learning algebra involves a lot of reinforcement. Thus we provide cumulative reviews at the end of every odd-numbered chapter starting with Chapter 3. Do these cumulative reviews after each chapter test, so that you are always refreshing your memory—making those neurons do their calisthenics. This way, studying for the final exam should be fairly easy.

4 How to Prepare for an Exam

The following steps are time-tested suggestions to help you prepare for an exam.

Step 1: Revisit your homework and the chapter review problems About one week before your exam, start to redo your homework assignments. If you don't understand a topic, seek out help. Work the problems in the chapter review as well. The problems are keyed to the section objectives. If you get a problem wrong, identify the objective and examples that illustrate the objective. Then review this material and try the problem in the chapter review again. If you get the problem wrong again, seek out help.

Step 2: Test yourself A day or two before the exam, take the chapter test under test conditions. Be sure to check your answers. If you got any problems wrong, determine why you got them wrong and remedy the situation.

Step 3: View the Chapter Test Prep Videos These videos show step-by-step solutions to the problems found in each of the book's chapter tests. Follow the worked-out solutions to any of the exercises on the chapter test that you want to study or review.

Step 4: Follow these rules as you train Be sure to arrive early at the location of the exam. Prepare your mind for the exam. Be sure you are well rested. Don't try to pull "all-nighters." If you need to study all night for an exam, then your time management is poor, and you should rethink how you are using your time or whether you have enough time set aside for the course.

Work Smart: Study Skills

Do not "cram" for an exam by pulling an "all-nighter."

1.1 Exercises

MyMathLab® MathXL PRACTICE

1. Why do you want to be successful in mathematics? Are your goals positive or negative? If you stated your goal negatively ("Just get me out of this course!"), can you restate it positively?
2. Name three activities in your daily life that involve the use of math (for instance, playing cards, operating your computer, or reading a credit-card bill).
3. What is your instructor's name?
4. What are your instructor's office hours? Where is your instructor's office?
5. What is your instructor's e-mail address?
6. Does your class have a website? Do you know how to access it? What information is located on the website?
7. Are there tutors available for this course? If so, where are they located? When are they available?
8. Name two other students in your class. What is their contact information? When can you meet with them to study?
9. List some of the things that you should do before class begins.
10. List some of the things that you should do during class.
11. List some of the things that you should do after class.
12. What is the point of the Chinese proverb quoted in this section?
13. What is the "readiness quiz"? How should it be used?
14. Name three features that appear in the margins. What is the purpose of each of them?
15. Name the categories of exercises that appear in this text.
16. How should the chapter review material be used?

17. How should the chapter test be used? What are the Chapter Test Prep Videos?

18. How should the cumulative review be used?

19. List the four steps that should be followed when preparing for an exam. Can you think of other methods of preparing for an exam that have worked for you?

20. Use the chart below to help manage your time. Be sure to fill in time allocated to various activities in your life, including school, work, and leisure.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
7 A.M.							
8 A.M.							
9 A.M.							
10 A.M.							
11 A.M.							
Noon							
1 P.M.							
2 P.M.							
3 P.M.							
4 P.M.							
5 P.M.							
6 P.M.							
7 P.M.							
8 P.M.							
9 P.M.							

1.2 Fractions, Decimals, and Percents

Objectives

1. Factor a Number as a Product of Prime Factors
2. Find the Least Common Multiple of Two or More Numbers
3. Write Equivalent Fractions
4. Write a Fraction in Lowest Terms
5. Round Decimals
6. Convert Between Fractions and Decimals
7. Convert Between Percents and Decimals

Work Smart

The first six primes are 2, 3, 5, 7, 11, and 13.

Work Smart: Study Skills

The icon ▶ means a video is available for this content. See page 5 for a description.

We base our discussion in this section on *natural numbers*. **Natural numbers** are the numbers 1, 2, 3, 4, and so on.

1 Factor a Number as a Product of Prime Factors

When we multiply, the numbers that are multiplied together are the **factors** and the answer is the **product.**

$$\underset{\text{factor}}{7} \cdot \underset{\text{factor}}{5} = \underset{\text{product}}{35}$$

When we write a number as a product, we say that we **factor** the number. For example, when we write 20 as the product $10 \cdot 2$, we say that we have factored 20.

Some natural numbers are *prime* numbers and others are *composite*.

Definition

A natural number is **prime** if its only factors are 1 and itself. Natural numbers that are not prime are called **composite.** The number 1 is neither prime nor composite.

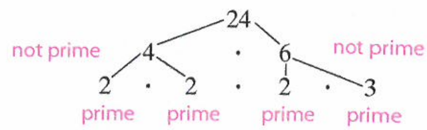
Examples of prime numbers are 2, 3, 5, 7, 11, and 13. When we write a composite number as the product of prime numbers, we say that we are writing the **prime**

factorization of the number. We can use a *factor tree* to find the prime factorization of a number. The process begins with finding two factors of the given number. Continue to factor until all factors are prime.

EXAMPLE 1 Finding the Prime Factorization

Write the prime factorization of 24.

Solution



Work Smart

We could have begun the factorization of 24 with the factors 8 and 3 instead of 4 and 6. Try this for yourself.

All the numbers in the last row are prime, so we are done. The prime factorization of 24 is $2 \cdot 2 \cdot 2 \cdot 3$. Order is not important in multiplying factors. The product could also be written as $3 \cdot 2 \cdot 2 \cdot 2$ or $2 \cdot 3 \cdot 2 \cdot 2$.

Quick ✓

1. A natural number is _____ if its only factors are 1 and itself.
2. In the statement $6 \cdot 8 = 48$, 6 and 8 are called _____ and 48 is called the _______.

In Problems 3–6, find the prime factorization of each number. If a number is prime, state so.

3. 12 **4.** 120

5. 31 **6.** 117

2 Find the Least Common Multiple of Two or More Numbers

A **multiple** of a number is the product of that number and any natural number. For example, the multiples of 2 are

$$2 \cdot 1 = 2,\quad 2 \cdot 2 = 4,\quad 2 \cdot 3 = 6,\quad 2 \cdot 4 = 8,\quad 2 \cdot 5 = 10,\quad 2 \cdot 6 = 12,\quad \text{and so on.}$$

Multiples of 3 are

$$3 \cdot 1 = 3,\quad 3 \cdot 2 = 6,\quad 3 \cdot 3 = 9,\quad 3 \cdot 4 = 12,\quad 3 \cdot 5 = 15,\quad 3 \cdot 6 = 18,\quad \text{and so on.}$$

Notice that the numbers 2 and 3 have 6 and 12 as common multiples. The smallest common multiple, called the *least common multiple*, of 2 and 3 is 6.

Definition

The **least common multiple (LCM)** of two or more natural numbers is the smallest number that is a multiple of each of the numbers.

For example, to find the least common multiple of 6 and 15, we could list the multiples of each number until we find the smallest common multiple, as follows:

Multiples of 6: 6, 12, 18, 24, 30, 36, 42, . . .
Multiples of 15: 15, 30, 45, 60, . . .

The least common multiple is 30. This approach works just fine for numbers, but it does not work for algebra. For this reason, we recommend that you follow the steps used in Example 2 to find the least common multiple so that you will be better prepared when we discuss the LCM again later in the course.

EXAMPLE 2 How to Find the Least Common Multiple

Find the least common multiple of 6 and 15.

Step-by-Step Solution

Step 1: Write each number as the product of prime factors, aligning common factors vertically.

Arrange the common factor of 3 in its own column:

$$\begin{aligned} 6 &= 2 \cdot 3 \\ 15 &= \phantom{2 \cdot{}} 3 \cdot 5 \end{aligned}$$

Step 2: Write down the common factor(s), if any. Then write down the remaining factors.

The common factor is 3.
The remaining factors are 2 and 5.

Step 3: Multiply the factors listed in Step 2. The product is the least common multiple (LCM).

The least common multiple of 6 and 15 is $2 \cdot 3 \cdot 5 = 30$

EXAMPLE 3 Finding the Least Common Multiple

Find the least common multiple of 18 and 15.

Solution

We first write each number as the product of prime factors.

Write the prime factors in each column that the numbers share, if any. Then write down the remaining factors. Find the product of the factors.

$$\begin{aligned} 18 &= 2 \cdot 3 \cdot 3 \\ 15 &= \phantom{2 \cdot{}} 3 \cdot \phantom{3 \cdot{}} 5 \\ & \ 2 \cdot 3 \cdot 3 \cdot 5 \end{aligned}$$

The LCM is $2 \cdot 3 \cdot 3 \cdot 5 = 90$.

Quick ✓

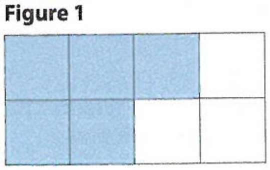
7. The ____ ______ ______ of two or more natural numbers is the smallest number that is a multiple of each of the numbers.

In Problems 8–11, find the LCM of the numbers.

8. 6 and 8
9. 45 and 72
10. 14 and 9
11. 12, 18, and 30

3 Write Equivalent Fractions

Figure 1



A fraction represents a part of a whole. For example, the fraction $\frac{5}{8}$ means "5 parts out of 8 parts." A fraction also indicates division: $\frac{5}{8}$ means "five divided by eight" and may be written as $8\overline{)5}$. Figure 1 shows the fraction $\frac{5}{8}$ visually.

In the fraction $\frac{5}{8}$, the number 5 is the **numerator** and the number 8 is the **denominator.** The denominator tells the number of equal parts that the whole is divided into, and the numerator tells the number of equal parts that are shaded. For example, in Figure 1 the box is divided into 8 equal parts, 5 of which are shaded.

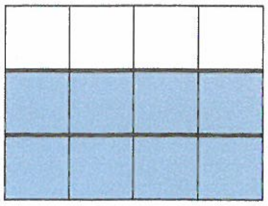
We use **whole numbers**, the natural numbers plus 0, for the numerator of a fraction, and we use natural numbers for the denominator.

Fractions without common denominators can be rewritten in equivalent forms so they have the same denominator.

Work Smart

The denominator of a number such as 7 is 1 because $7 = \frac{7}{1}$.

Figure 2



Definition

Equivalent fractions are fractions that represent the same part of a whole.

For example, $\frac{2}{3}$ and $\frac{8}{12}$ are equivalent fractions. To understand why, consider Figure 2. Break the whole into 12 parts and shade 8 of these parts, so that the shaded region represents $\frac{8}{12}$ of the rectangle. If we consider only the 3 parts separated by the thick black lines, we can see that 2 parts are shaded for a fraction of $\frac{2}{3}$. In each case, the same portion of the rectangle is shaded, so $\frac{2}{3}$ and $\frac{8}{12}$ are equivalent fractions.

In Words

We can obtain an equivalent fraction by multiplying the numerator and denominator of the fraction by the same nonzero number.

How do we obtain equivalent fractions? The answer lies in the following property.

If a, b, and c are whole numbers, then

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c} \quad \text{if } b \neq 0, c \neq 0$$

EXAMPLE 4 Writing an Equivalent Fraction

Write the fraction $\frac{3}{4}$ as an equivalent fraction with a denominator of 20.

Solution

We want to know $\frac{3}{4}$ equals "what" over 20, or $\frac{3}{4} = \frac{?}{20}$. To write $\frac{3}{4}$ with a denominator of 20, we multiply the numerator and denominator of $\frac{3}{4}$ by 5. Do you see why?

$$\frac{3}{4} = \frac{3 \cdot 5}{4 \cdot 5}$$
$$= \frac{15}{20}$$

Quick ✓

12. In the fraction $\frac{7}{12}$, 7 is called the ________ and 12 is called the ________.

13. Fractions that represent the same portion of a whole are called ________ ________.

In Problems 14 and 15, rewrite each fraction with the denominator indicated.

14. $\frac{1}{2}$; 10 **15.** $\frac{5}{8}$; 48

We sometimes need to rewrite two or more fractions so that they both have the same denominator. For example, we could rewrite the fractions $\frac{5}{6}$ and $\frac{3}{8}$ with a common denominator of 24, 48, 96 and so on because these are common multiples of the denominators 6 and 8. Notice that 24 is the least common multiple of 6 and 8.

Definition

The **least common denominator (LCD)** is the least common multiple of the denominators of a group of fractions.

EXAMPLE 5 How to Write Two Fractions as Equivalent Fractions with the LCD

Write $\frac{5}{8}$ and $\frac{9}{20}$ as equivalent fractions with the least common denominator.

Step-by-Step Solution

Step 1: Find the least common denominator of the fractions.

The denominators of $\frac{5}{8}$ and $\frac{9}{20}$ are 8 and 20.

Write each denominator as the product of prime factors: $8 = 2 \cdot 2 \cdot 2$

$20 = 2 \cdot 2 \cdot 5$

Write the common factors; then write the uncommon factors: $\text{LCD} = 2 \cdot 2 \cdot 2 \cdot 5$

$= 40$

Step 2: Rewrite each fraction with the least common denominator.

Multiply the numerator and denominator of $\frac{5}{8}$ by 5: $\frac{5}{8} = \frac{5 \cdot 5}{8 \cdot 5}$

$= \frac{25}{40}$

Multiply the numerator and denominator of $\frac{9}{20}$ by 2: $\frac{9}{20} = \frac{9 \cdot 2}{20 \cdot 2}$

$= \frac{18}{40}$

Quick ✓

16. The ____ ______ ________ is the least common multiple of the denominators of a group of fractions.

In Problems 17 and 18, write the equivalent fractions with the least common denominator.

17. $\frac{1}{4}$ and $\frac{5}{6}$

18. $\frac{9}{20}$ and $\frac{11}{16}$

4 Write a Fraction in Lowest Terms

Definition

A fraction is written in **lowest terms** if the numerator and the denominator share no common factor other than 1.

In Words

To write a fraction in lowest terms, find any common factors between the numerator and denominator, and divide out the common factors.

We can write fractions in lowest terms using the fact that

$$\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$$

Thus, to write a fraction in lowest terms, we write the numerator and the denominator as a product of primes and then divide out common factors.

EXAMPLE 6 Writing a Fraction in Lowest Terms

Write $\frac{24}{40}$ in lowest terms.

Solution

Write the numerator and the denominator as the product of primes and divide out common factors.

Work Smart

Use different slash marks to keep track of factors that have divided out. Also, we may use nonprime factors when writing a fraction in lowest terms. In Example 6 we could write

$$\frac{24}{40} = \frac{8 \cdot 3}{8 \cdot 5} = \frac{3}{5}$$

$$\frac{24}{40} = \frac{2 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 5}$$

Divide out common factors: $= \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 3}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 5}$

$$= \frac{3}{5}$$

Quick ✓

19. A fraction is written in ______ ______ if the numerator and the denominator share no common factor other than 1.

In Problems 20–22, write each fraction in lowest terms.

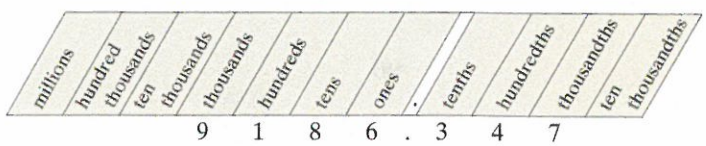
20. $\frac{45}{80}$ **21.** $\frac{4}{9}$ **22.** $\frac{16}{56}$

5 Round Decimals

Decimals and percentages commonly occur in everyday life. You receive a 92% on your test, there is a 10% discount on jeans, we pay 7.75% in sales tax, 45% of the people polled support a proposition. Before we discuss decimals and percents, we consider place value.

Figure 3 shows how we interpret the place value of each digit in the number 9186.347. For example, the 7 is in the thousandths position, 3 is in the tenths position, and the 8 is in the tens position.

Figure 3



The number 9186.347 is read "nine thousand, one hundred eighty-six and three hundred forty-seven thousandths."

Quick ✓

In Problems 23–26, tell the place value of the digit in the given number.

23. 235.71; the 1

24. 56,701.28; the 2

25. 278,403.95; the 8

26. 0.189; the 9

We round decimals in the same way we round whole numbers. First, identify the specified place value in the decimal. If the digit to the right is 5 or more, add 1 to the digit; if the digit to the right is 4 or less, leave the digit as it is. Then drop the digits to the right of the specified place value.

EXAMPLE 7 Rounding a Decimal Number

(a) Round 8.726 to the nearest hundredth.

(b) Round 0.9451 to the nearest thousandth.

Solution

(a) To round to the nearest hundredth, we determine that 2 is in the hundredths place: 8.726. The number to the right of 2 is 6. Since 6 is greater than 5, we round 8.726 to 8.73.

(b) To round 0.9451 to the nearest thousandth, we see that 5 is in the thousandths place: 0.9451. The number to the right of 5 is 1. Since 1 is less than 5, we round 0.9451 to 0.945.

Quick ✓

In Problems 27–31, round each number to the given decimal place.

27. 0.173 to the nearest tenth

28. 0.932 to the nearest hundredth

29. 1.396 to the nearest hundredth

30. 690.004 to the nearest hundredth

31. 59.98 to the nearest tenth

6 Convert Between Fractions and Decimals

Convert a Fraction to a Decimal

To convert a fraction to a decimal, divide the numerator of the fraction by the denominator of the fraction until the remainder is 0 or the remainder repeats.

EXAMPLE 8 Converting a Fraction to a Decimal

Convert each number to a decimal.

(a) $\frac{9}{20}$ **(b)** $\frac{2}{3}$

Solution

(a)

$$\frac{9}{20} = 20\overline{)9.00} = 0.45$$

```
      0.45
20)9.00
   8 0
   100
   100
     0
```

Therefore, $\frac{9}{20} = 0.45$.

(b)

```
      0.666
 3)2.000
   1 8
     20
     18
      20
      18
       2
```

$$\frac{2}{3} = 3\overline{)2.000}$$

Notice that the remainder, 2, repeats. So $\frac{2}{3} = 0.666\ldots$.

In Example 8(a), the decimal 0.45 is called a **terminating decimal** because the decimal stops after the 5. In Example 8(b), the number 0.666 . . . is called a **repeating decimal** because the 6 repeats indefinitely. The decimal 0.666 . . . can also be written as $0.\overline{6}$. The bar over the 6 means the 6 repeats.

Quick ✓

In Problems 32–35, write the fraction as a decimal.

32. $\frac{2}{5}$

33. $\frac{5}{6}$

34. $\frac{11}{8}$

35. $\frac{3}{7}$

Based on Examples 8(a) and (b) and Quick Check Problems 32–35, you should notice that **every fraction has a decimal representation that either terminates or repeats.**

Convert a Decimal to a Fraction

To convert a decimal to a fraction, identify the place value of the last digit in the decimal. Write the decimal as a fraction using the place value of the last digit as the denominator, and write in lowest terms.

EXAMPLE 9 Writing a Decimal as a Fraction

Convert each decimal to a fraction and write in lowest terms.

(a) 0.8 **(b)** 0.77 **(c)** 4.237

Solution

(a) 0.8 is equivalent to 8 tenths, or $\frac{8}{10}$. Because $\frac{8}{10} = \frac{4 \cdot 2}{5 \cdot 2} = \frac{4}{5}$, we write $0.8 = \frac{4}{5}$.

(b) 0.77 is equivalent to 77 hundredths, or $\frac{77}{100}$.

(c) 4.237 is equivalent to 4237 thousandths, or $\frac{4237}{1000}$.

Quick ✓

In Problems 36–38, write the decimal as a fraction and write in lowest terms.

36. 0.6 **37.** 0.17 **38.** 0.625

7 Convert Between Percents and Decimals

When computing with percents, it is convenient to write percents as decimals. How is a percent converted to a decimal? Let's see.

Convert a Percent to a Decimal

Definition

The word **percent** means **parts per hundred** or **parts out of one hundred.**

So 25% means 25 parts out of 100 parts. Therefore, $25\% = \frac{25}{100} = \frac{1 \cdot 25}{4 \cdot 25} = \frac{1}{4}$.

Since the word percent means "parts per hundred," 100% means "100 parts per 100," so $100\% = 1$. Therefore, to convert from a percent to a decimal, multiply the percent by $\frac{1}{100\%}$.

EXAMPLE 10 Writing a Percent as a Decimal

Write the following percents as decimals:

(a) 27% **(b)** 150%

Solution

(a)
$$27\% = 27\% \cdot \frac{1}{100\%} = \frac{27}{100} = 0.27$$

(b)
$$150\% = 150\% \cdot \frac{1}{100\%} = \frac{150}{100} = 1.5$$

Work Smart

To convert from a percent to a decimal, move the decimal point two places to the left and drop the % symbol.

Quick ✓

39. The word percent means parts per ______, so 35% means __ parts out of 100 parts or $\frac{\_\_}{100}$.

In Problems 40–43, write the percent as a decimal.

40. 23% **41.** 1%

42. 72.4% **43.** 127%

Convert a Decimal to a Percent

Because 100% = 1, to convert a decimal to a percent, multiply the decimal by $\frac{100\%}{1}$.

EXAMPLE 11 Writing a Decimal as a Percent

Write the following decimals in percent form:

(a) 0.445 **(b)** 1.42

Solution

(a) $0.445 = 0.445 \cdot \frac{100\%}{1}$

$= 44.5\%$

(b) $1.42 = 1.42 \cdot \frac{100\%}{1}$

$= 142\%$

Work Smart

To convert from a decimal to a percent, move the decimal point two places to the right and add the % symbol.

Quick ✓

44. To convert a decimal to a percent, multiply the decimal by _____.

In Problems 45–48, write the decimal as a percent.

45. 0.15 **46.** 0.8

47. 1.372 **48.** 0.004

1.2 Exercises

MyMathLab® MathXL® PRACTICE

Exercise numbers in green have complete video solutions in MyMathLab.

Problems **1–48** *are the* Quick ✓s *that follow the* EXAMPLES.

Building Skills

In Problems 49–62, find the prime factorization of each number. If a number is prime, state so. See Objective 1.

49. 25 **50.** 9 **51.** 28

52. 100 **53.** 21 **54.** 35

55. 36 **56.** 54 **57.** 50

58. 70 **59.** 53 **60.** 79

61. 252 **62.** 315

In Problems 63–74, find the LCM of each set of numbers. See Objective 2.

63. 6 and 21 **64.** 10 and 14

65. 18 and 63 **66.** 42 and 18

67. 15 and 14 **68.** 55 and 6

69. 30 and 45 **70.** 8 and 60

71. 5, 6, and 12 **72.** 9, 15, and 20

73. 3, 8, and 9 **74.** 4, 18, and 20

In Problems 75–80, write each fraction with the given denominator. See Objective 3.

75. Write $\frac{2}{3}$ with denominator 12.

76. Write $\frac{4}{5}$ with denominator 15.

77. Write $\frac{3}{4}$ with denominator 24.

78. Write $\frac{5}{14}$ with denominator 28.

79. Write 7 with denominator 3.

80. Write 4 with denominator 10.

In Problems 81–88, write the equivalent fractions with the least common denominator. See Objective 3.

81. $\frac{1}{2}$ and $\frac{3}{8}$

82. $\frac{3}{4}$ and $\frac{5}{12}$

83. $\frac{3}{5}$ and $\frac{2}{3}$

84. $\frac{1}{4}$ and $\frac{2}{9}$

85. $\frac{1}{12}$ and $\frac{5}{18}$

86. $\frac{5}{12}$ and $\frac{7}{15}$

87. $\frac{2}{9}$ and $\frac{7}{18}$ and $\frac{7}{30}$

88. $\frac{7}{10}$ and $\frac{1}{4}$ and $\frac{5}{6}$

In Problems 89–96, write each fraction in lowest terms. See Objective 4.

89. $\frac{14}{21}$

90. $\frac{9}{15}$

91. $\frac{38}{18}$

92. $\frac{81}{36}$

93. $\frac{22}{66}$

94. $\frac{9}{27}$

95. $\frac{18}{3}$

96. $\frac{36}{4}$

In Problems 97–102, tell the place value of the indicated digit in the given number. See Objective 5.

97. 3465.902; the 0

98. 549,813.0267; the 8

99. 357.469; the 5

100. 9124.786; the 7

101. 2018.3764; the 6

102. 539.016; the 9

In Problems 103–110, round each number to the given place. See Objective 5.

103. 578.206 to the nearest tenth

104. 7298.0845 to the nearest hundredth

105. 354.678 to the nearest hundredth

106. 543.56 to the nearest whole number

107. 3682.0098 to the nearest thousandth

108. 683.098 to the nearest hundredth

109. 29.96 to the nearest whole number

110. 37.999 to the nearest tenth

In Problems 111–120, convert each fraction to a decimal. See Objective 6.

111. $\frac{5}{8}$

112. $\frac{3}{4}$

113. $\frac{2}{7}$

114. $\frac{2}{9}$

115. $\frac{5}{16}$

116. $\frac{11}{32}$

117. $\frac{3}{13}$

118. $\frac{6}{13}$

119. $\frac{29}{25}$

120. $\frac{57}{50}$

In Problems 121–126, write each decimal as a fraction in lowest terms. See Objective 6.

121. 0.75

122. 0.25

123. 0.5

124. 0.4

125. 0.982

126. 0.358

In Problems 127–132, write each percent as a decimal. See Objective 7.

127. 37%

128. 59%

129. 6.02%

130. 8.25%

131. 0.1%

132. 0.5%

In Problems 133–138, write each decimal as a percent. See Objective 7.

133. 0.2

134. 0.5

135. 0.275

136. 0.349

137. 2

138. 1

Mixed Practice

In Problems 139–144, write each fraction as a decimal, rounded to the indicated place.

139. $\frac{13}{6}$ to the nearest tenth

140. $\frac{15}{8}$ to the nearest tenth

141. $\frac{8}{3}$ to the nearest hundredth

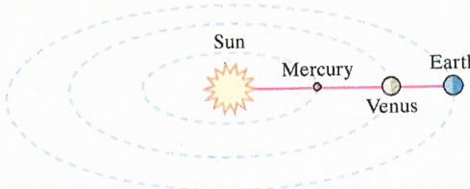
142. $\frac{9}{7}$ to the nearest hundredth

143. $\frac{14}{27}$ to the nearest thousandth

144. $\frac{18}{31}$ to the nearest thousandth

Applying the Concepts

145. Planets in Our Solar System At a certain point, Mercury, Venus, and Earth lie on a straight line. If it takes these planets 3, 7, and 12 months, respectively, to revolve around the sun, what is the fewest number of months until they align this way again?



146. Talladega Raceway At Talladega, one of the crew chiefs discovered that in a given time interval, Jeff Gordon completed 21 laps, Dale Earnhardt Jr. completed 18 laps, and Robby Gordon completed 15 laps. Suppose all three drivers begin at the same time. How many laps would need to be completed so that all three drivers were at the finish line at exactly the same time?

147. Sam's Medication Bob gives his dog Sam one type of medication every 4 days, and a second type of medication every 10 days. How often does Bob give Sam both medications on the same day?

148. Visiting Columbus Pamela and Geoff both visit Columbus on business. Pamela flies to Columbus from Atlanta every 14 days, and Geoff takes the train to Columbus from Cincinnati every 20 days. How often are both Pamela and Geoff in Columbus on business?

149. Survey Data In a survey of 500 students, 325 stated that they work at least 25 hours per week. Express the fraction of students that work at least 25 hours per week as a fraction in lowest terms.

150. Survey Data In a survey of 750 students, 450 stated that they are enrolled in 15 or more semester hours. Express the fraction of students who are enrolled in 15 or more semester hours as a fraction in lowest terms.

In Problems 151–158, express answers to the nearest hundredth of a percent, if necessary.

151. Eating Healthy? In a poll conducted by Zogby International of 1200 adult Americans, 840 stated that they believe that they eat healthy foods. What percentage of adult Americans believe that they eat healthy foods?

152. Ghosts In a survey of 1100 adult women conducted by Harris Interactive, it was determined that 640 believe in ghosts. What percentage of adult women believe in ghosts?

153. Test Score A student earns 85 points out of a total of 110 points on an exam. Express this score as a percent.

154. Test Score A student earns 80 points out of a total of 115 points on an exam. Express this score as a percent.

155. Time Utilization In a 24-hour day, Jackson sleeps for 8 hours, works for 4 hours, and goes to school and studies for 6 hours.

(a) What percent of the time does Jackson sleep?

(b) What percent of the time does Jackson work?

(c) What percent of the time does Jackson go to school and study?

156. Tree Inventory An arborist counted the number of ash trees in Whetstone Park. He found that there were 48 white ash trees, 51 green ash trees, and 2 blue ash trees.

(a) What percent of the ash trees in Whetstone Park were white ashes?

(b) What percent of the ash trees in Whetstone Park were green ashes?

(c) What percent of the ash trees in Whetstone Park were blue ashes?

157. Cashews A single serving of cashews contains 14 grams of fat. Of this, 3 grams is saturated fat. What percentage of fat grams is saturated fat in a single serving of cashews?

158. Cheese Pizza A single serving of cheese pizza contains 11 grams of fat. Of this, 5 grams are saturated fat. What percentage of fat grams is saturated fat in a single serving of cheese pizza?

Extending the Concepts

159. The Sieve of Eratosthenes Eratosthenes (276 B.C.–194 B.C.) was born in Cyrene, which is now in Libya in North Africa. He devised an algorithm (a series of steps that are followed to solve a problem) for identifying prime numbers. The algorithm works as follows.

Step 1: List all the natural numbers that are greater than or equal to 2.

Step 2: The first number in the list, 2, is prime. Cross out all multiples of 2. For example, cross out 2, 4, 6, ...

Step 3: Identify the next number in the list after the most recently identified prime number. For example, we already know 2 is a prime number, so the next number in the list, 3, is also prime. Cross out all multiples of this number.

Step 4: Repeat Step 3.

Use the algorithm to find all the prime numbers less than 100.